

# C.U.SHAH UNIVERSITY

## Summer Examination-2016

Subject Name : Engineering Mathematics-I

Subject Code : 4TE01EMT2

Branch : B.Tech (All)

Semester : 1

Date :21/04/2016

Time :10:30 To 1:30

Marks : 70

Instructions:

- (1) Use of Programmable calculator & any other electronic instrument is prohibited.
- (2) Instructions written on main answer book are strictly to be obeyed.
- (3) Draw neat diagrams and figures (if necessary) at right places.
- (4) Assume suitable data if needed.

Q-1

Attempt the following questions:

(14)

- a) The polar form of complex number  $\frac{1+i}{1-i}$ .
- a)  $\cos \frac{\pi}{2} + i \sin \frac{\pi}{2}$     b)  $\sin \frac{\pi}{2} + i \cos \frac{\pi}{2}$     c)  $\cos \frac{\pi}{4} + i \sin \frac{\pi}{4}$     d)  $\sin \frac{\pi}{4} + i \cos \frac{\pi}{4}$
- b) The Imaginary part of Complex number  $e^{3z}$  is
- a)  $e^y \sin x$     b)  $e^x \cos y$     c)  $e^{3x} \cos 3y$     d)  $e^{3x} \sin 3y$
- c)  $\lim_{x \rightarrow 0} \frac{\tan x}{x} = \underline{\hspace{2cm}}$ .
- a) 0    b) 1    c)  $\infty$     d) -1
- d)  $\lim_{x \rightarrow 0} \frac{x - \sin x}{x} = \underline{\hspace{2cm}}$ .
- a) 0    b) 1    c)  $\infty$     d) -1
- e) The rank of diagonal matrix  $\begin{bmatrix} 1 & 0 & 0 \\ 0 & -2 & 0 \\ 0 & 0 & 3 \end{bmatrix}$  is
- a) 0    b) 1    c) 3    d) -2
- f) If the rank of the matrix  $\begin{bmatrix} 1 & 0 & 0 \\ 0 & -2 & 0 \\ 0 & 0 & 3 \end{bmatrix}$  is 3, then x is not equal to
- a) 3    b) 4    c) 5    d) 6
- g) If the power of x & y both are even, then the curve is symmetrical about
- b) X-axis    b) Y-axis    c) about both X & Y axes    d) None of these
- h) If the two tangents at the point are real & coincide, the double point is called
- a) a node    b) a cusp    c) a conjugate point    d) None of these
- i) The series  $1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \dots$  represents expansion of



- a)  $\sin x$       b)  $\cos x$       c)  $\cosh x$       d)  $\sinh x$
- j)** If  $y = \cos^{-1} x$ , then  $x = \dots$
- a)  $1 - \frac{y^2}{2!} + \frac{y^4}{4!} - \dots$     b)  $1 + \frac{y^2}{2!} + \frac{y^4}{4!} + \dots$     c)  $y - \frac{y^3}{3!} + \frac{y^5}{5!} - \dots$     d) None of these
- k)** If  $u = \sin^{-1} \left( \frac{x^2 + y^2}{x + y} \right)$  then  $x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y} = \dots$
- a)  $u$                       b)  $2u$                       c)  $\tan u$                       d)  $\sin u$
- l)** If  $p = r \tan \theta$  then  $\frac{\partial p}{\partial r}$  is
- a)  $\tan \theta + r \sec^2 \theta$                       b)  $\sec^2 \theta$                       c)  $\tan \theta$                       d) None of these
- m)** If  $x = r \cos \theta, y = r \sin \theta, z = z$  then  $\frac{\partial(x, y, z)}{\partial(r, \theta, z)} = \dots$
- a)  $\frac{1}{r}$                       b)  $r^2 \sin \theta$                       c)  $r$                       d)  $r^2 \cos \theta$
- n)** If  $\frac{\partial(u, v)}{\partial(x, y)} * \frac{\partial(x, y)}{\partial(u, v)} = \dots$
- a)  $0$                       b)  $-1$                       c)  $1$                       d) None of these

**Attempt any four questions from Q-2 to Q-8**

**Q-2                      Attempt all questions                      (14)**

**A**                      i) Find modulus and principal argument of  $z = \frac{1-7i}{(3+4i)}$ .                      **(03)**

ii) Simplify  $\frac{(\cos 3\theta + i \sin 3\theta)^8 (\cos \theta - i \sin \theta)^5}{(\cos 2\theta + i \sin 2\theta)^{-2} (\cos 5\theta - i \sin 5\theta)^{-3}}$                       **(04)**

**B**                      (i) Find and plot the fourth root of unity on the circle.                      **(07)**

**Q-3                      Attempt all questions                      (14)**

**A**                      i) Evaluate:  $\lim_{x \rightarrow 0} \left[ \frac{1^x + 2^x + 3^x}{3} \right]^{\frac{1}{x}}$ .                      **(04)**

ii) Find Maclaurin's Series of  $f(x) = \sin x$ .                      **(03)**

**B**                      i) Show that  $f(x, y) = \begin{cases} \frac{xy}{x^2 + y^2}; & (x, y) \neq (0, 0) \\ 0 & ; (x, y) = (0, 0) \end{cases}$  is continuous at every point                      **(04)**

except at the origin.

ii) Evaluate:  $\lim_{x \rightarrow 0} (\cos x)^{\cot x} \dots$                       **(03)**

**Q-4                      Attempt all questions                      (14)**

**A**                      Trace the curve (Cissoid of Diocle)  $y^2(2a - x) = x^3$ .                      **(07)**



**B** If  $y = \frac{x}{x^2 + a^2}$ . find  $y_n$ . (07)

**Q-5** Attempt all questions (14)

**A** Find the Eigen Values & Eigen Vectors of the Matrix  $A = \begin{bmatrix} 11 & -4 & -7 \\ 7 & -2 & -5 \\ 10 & -4 & -6 \end{bmatrix}$  (07)

**B** State Caley Hamilton Theorem . Verify Caley Hamilton Theorem for the matrix  $A = \begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$  & Hence, find  $A^{-1}$  (07)

**Q-6** Attempt all questions (14)

**A** (i) Find the rank of matrix  $A = \begin{bmatrix} 2 & 3 & 4 & 5 \\ 3 & 4 & 5 & 6 \\ 4 & 5 & 6 & 7 \\ 9 & 10 & 11 & 12 \end{bmatrix}$  (03)

(ii) Find the inverse of the matrix  $A = \begin{bmatrix} -1 & 1 & 2 \\ 3 & -1 & 1 \\ -1 & 3 & 4 \end{bmatrix}$  using Guass Jordan Method (04)

**B**  $x + y + z = 6$  (04)

(i) Check the consistency of the system  $x + 2y + 3z = 14$  .

$$x + 4y + 9z = 36$$

(ii) Check the following set of vectors  $(1,0,1), (1,1,0), (1,-1,1), (1,2,-3)$  is Linearly Independent or dependent ? (03)

**Q-7** Attempt all questions (14)

**A** (i) If  $u = \log(x^3 + y^3 - x^2y - xy^2)$ . then Prove that  $\left(\frac{\partial}{\partial x} + \frac{\partial}{\partial y}\right)^2 u = -\frac{4}{(x+y)^2}$ . (05)

(ii) Find the values of  $\frac{\partial f}{\partial x}$  &  $\frac{\partial f}{\partial y}$  at the point  $(4,-5)$ . if  $f(x, y) = x^2 + 3xy + y - 1$ . (02)

**B** (i) If  $u = \sin^{-1}\left(\frac{x+y}{\sqrt{x} + \sqrt{y}}\right)$ . Prove that (07)

a)  $x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y} = \frac{1}{2} \tan u$

b)  $x^2 \frac{\partial^2 u}{\partial x^2} + 2xy \frac{\partial^2 u}{\partial x \partial y} + y^2 \frac{\partial^2 u}{\partial y^2} = \frac{1}{4} (\tan^3 u - \tan u)$ .

**Q-8** Attempt all questions (14)

**A** (i) Find Maxima & Minima of the function  $x^3 + y^3 - 3x - 12y + 20$ . (05)



(ii) If  $x = r \cos \theta, y = \sin \theta$  then find  $\frac{\partial(x, y)}{\partial(r, \theta)}$ .

**B**

(i) Find the Taylor's series expansion of  $f(x) = \tan^{-1}\left(\frac{y}{x}\right)$  in powers of  $(x-1)$

**(02)**

**(05)**

**(02)**

&  $(y-1)$ .

(ii) Find the equations of tangent plane & normal line at the point  $(-2, 2, -3)$  to the

ellipsoid  $\frac{x^2}{4} + y^2 + \frac{z^2}{9} - 3 = 0$

